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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

REMARKS ON NO. 99 (MARCH, 1899; MARCH, 1900; APRIL, 1904).

I. By **DR. F. H. SAFFORD**, University of Pennsylvania.

The following method for the six-problem appears to be exhaustive, and gives two solutions which appear are independent.

Consider only those arrangements which contain the natural order 123456 as the first line. The only admissible sets containing 124 are 124365, 124635, 124653. This gives only three possible second lines. With each of these pairs of lines a certain number of lines containing 125 may be found which will not conflict with the preceding. This method gives finally six sets which are the only ones possible, viz.,

A	*B*	*C*	*D*	*E*	*F*
123456	123456	123456	123456	123456	123456
124365	124635	124365	215634	124653	124365
125463	125643	125643	214536	125364	125643
126534	126534	126534	615423	126435	126534
134625	152436	245163	614352	156243	245316
245163	245136	241536	642513	143625	241635
241536	241563	246315	316452	245163	246153
246135	162354	164523	125643	241365	463251
162354	164523	162354	146235	162354	625413
465231	625314	253146	314265	164523	623154

Of these sets at most two are independent. *C* is the one due to Dr. Judson. The other five are convertible into each other in *various* ways. *E* is the one recently given by Dr. Dickson.

Adding 1 to each element of *A*, but noticing that 6 becomes 1, we obtain *B*. Similarly treating *B*, we get *D*.

In *A* change 5 to 3, 3 to 6, 6 to 5, and the result is *E*. The addition of 1 to the elements of *E* gives *F*, thus completing the proof of dependence of *A*, *B*, *D*, *E*, *F*. *C* may be written thus:

123456	245163	241536
126534	253146	251364
124365	236154	231645
125643	264135	261453

Each set of four will be reproduced by changing 3 to 6, 6 to 4, 4 to 5, 5 to 3. Also each set of four may be written in two sets of two each, using alternate lines, and now each set of two is reproduced by changing 3 to 4, 4 to 3, 5 to 6, 6 to 5. The other sets possess similar properties.

II. By DR. L. E. DICKSON, The University of Chicago.

The independence of Dr. Judson's solution of the six-problem,

$$\text{(J)}\quad \begin{array}{lllll} ABCDEF, & ABDCFE, & ABEFDC, & ABFECD, & ACFDBE, \\ ACEBFD, & ACBEDF, & ADECBF, & ADBFCE, & AEDBCF, \end{array}$$

from the solutions obtained by Dr. Safford and myself may be established most satisfactorily by group-theory.* Since $ABCDEF$ and $AFEDCB$ are regarded to form the same seating at a round table, there are 20 arrangements of the 6 letters each beginning with A. The substitutions by which they are derived from the natural order $ABCDEF$ are the following:

Identity, $(BF)(CE)$; $(CD)(EF)$, $(BEDCF)$; $(CEDF)$, $(BCDF)$; $(CFDE)$, $(BDEF)$; $(BCFE)$, $(BEFC)$; $(BCEFD)$, $(BD)(CF)$; $(BC)(DE)$, $(BFCDE)$; $(BDCE)$, $(BFDC)$; $(BDFEC)$, $(BE)(DF)$; $(BECD)$, $(BFED)$;

those of each pair leading to the same seating. These 20 substitutions form a *group*, the Lagrange group $(BCEFD)_{20}$. By cyclic interchange of A, B, C, D, E, F, we may write the 10 arrangements (J) so that they all begin with any chosen letter instead of A. There will result a group of order 20 leaving fixed this chosen letter. The simplest proof follows from the remark that $(ABCDEF)$ transforms the set (J) into itself, giving rise to the permutation (1), (2, 8, 3, 10, 4, 7)(5, 9, 6) of the arrangements.

If a new solution (S) is not independent of (J), there exists a substitution on A, ..., F which transforms (J) into (S), so that for (S) also there will be a *group* of order 20 whose pairs of substitutions replace one arrangement by the other nine, each written to begin with any chosen letter. Now for my solution

$$\begin{array}{lllll} ABCDEF, & ABDFEC, & ABECFD, & ABFDCE, & ACDBFE, \\ ACFEBD, & ACBEDF, & ADECBF, & ADCFBE, & AEDBCF, \end{array}$$

i. e., "E" in Dr. Safford's list above, the 20 substitutions are

Identity, $(BF)(CE)$; (CDF), $(BCEFD)$; $(CEFD)$, $(BDCF)$; (CFE), (BEF); $(BCD)(EF)$, $(BED)(CF)$; $(BCFDE)$, $(BDEFC)$; $(BC)(DE)$, $(BFCDE)$; $(BDCE)$, $(BFDC)$; $(BDFE)$, $(BEC)(DF)$; $(BECD)$, $(BFED)$,

and do not form a group since the inverse of (CDF) is not in the set.

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*Group-theory accounts for the properties noted at the end of Dr. Safford's remarks.